EXCELENCIA SEVERO OCHOA

ICMAT INSTITUTO DE CIENCIAS MATEMÁTICAS

Real Harmonic Analysis and its Applications to Partial Differential Equations and Geometric Measure Theory

On the occasion of the 60th birthday of Steve Hofmann

ICMAT (Madrid, Spain)

May 28 - June 1, 2018

MINISTERIO DE ECONOMÍA, INDUSTRIA Y COMPETITIVIDAD

EXCELENCIA SEVERO OCHOA

CSIC CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS

erc

NSF

Abstracts Short Talks

Akman, Murat (University of Connecticut, USA)

Small perturbations of elliptic operators in uniform domains satisfying the CDC

Abstract: In this talk, we study small perturbations of elliptic operators in uniform domains satisfying the capacity density condition (CDC). In particular, we focus on the following problem: suppose that we have "good estimates" for the Dirichlet problem for a uniformly elliptic operator L_0, under what optimal conditions, are those good estimates transferred to the Dirichlet problem for uniformly elliptic operator L which is a perturbation of L_0? We prove that if discrepancy between L_0 and L satisfies certain smallness assumption then the elliptic measure ω_L corresponding to L is in the reverse Hölder class with exponent 2 with respect to the elliptic measure ω_{L_0} corresponding to L_0.

Our work extends the classical result of Dahlberg and Fefferman, Kenig, and Pipher in Lipschitz domains, and Milakis, Pipher, and Toro in chord-arc domains to uniform domains satisfying the CDC.

This is a joint work in progress with Steve Hofmann, José María Martell, and Tatiana Toro.

Aldaz, Jesús M. (UAM-ICMAT, Spain)

Boundedness of averaging operators on geometrically doubling metric spaces

Abstract: We prove that averaging operators are uniformly bounded on L^1 for all geometrically doubling metric measure spaces, with bounds independent of the measure. From this result, the L^1 convergence of averages as $r \to 0$ immediately follows.

Brocchi, Gianmarco (University of Birmingham, UK)

Existence of extremizers for a Strichartz estimate for the fourth order Schrödinger equation

Abstract: In dispersive PDE, Strichartz estimates are a fundamental tool in understanding the evolution of waves. The search for extremizers in the corresponding inequalities is an active area of research, and is intimately related with the study of the Fourier extension operator from certain hypersurfaces.

In this talk, we discuss the sharp Strichartz estimate

$$|| \, |\partial_x|^{\frac{1}{3}} e^{it\partial_x^4} f||_{L^6_{t,x}(\mathbb{R}^{1+1})} \leq \mathbf{S}||f||_{L^2(\mathbb{R})}$$

for the fourth order Schrödinger equation in one spatial dimension

$$i\partial_t u + \partial_x^4 u = 0.$$

A careful analysis of the convolution measure on the quartic shows that extremizers for this inequality do exist, resolving the dichotomy in (2).

References

(1) Gianmarco Brocchi, Diogo Oliveira e Silva, and René Quilodrán, *Sharp Strichartz estimates for fractional Schrödinger equations*, Preprint.

(2) Jin-Cheng Jiang, Benoit Pausader, and Shuanglin Shao, *The linear profile decomposition for the fourth order Schrödinger equation*, Elsevier, Journal of Differential Equations, 2010.

Cavero de Carondelet, Juan (ICMAT, Spain)

Perturbations of elliptic operators and Carleson measure estimates

Abstract: We study the relation between the boundary behavior of a class of solutions to the Dirichlet problem and the absolute continuity of the elliptic measure with respect to surface measure, in a quantitative fashion. We extend a theorem of Kenig-Kirchheim-Pipher-Toro to the setting of 1-sided chord-arc domains: If every bounded weak solution of L satisfies a Carleson measure estimate, then the elliptic measure $\omega_L \in A_\infty(\partial\Omega)$. This allows us to extend the perturbation result of Fefferman-Kenig-Pipher and Milakis-Pipher-Toro to non-symmetric real elliptic operators in 1-sided chord-arc domains. Moreover, we obtain that $\omega_L \in A_\infty(\partial\Omega)$ if and only if $\omega_{L^\top} \in A_\infty(\partial\Omega)$, where $L^\top$ is the transpose of L, assuming some Carleson condition on the derivatives of the coefficients. This talk is part of a joint work with José M. Martell, Steve Hofmann and Tatiana Toro.

Chua, Seng-Kee (National University of Singapore, Singapore)

Global Subrepresentation Formulas in Chain Domains with Irregular Boundaries

Abstract: In chain domains with rough boundaries, we derive global subrepresentation formulas (i.e., pointwise inequalities reminscent of part of the Fundamental Theorem of Calculus) and show how they lead to global first order Poincaré-Sobolev estimates in these domains. Domains considered in this talk include Φ-John domains that are generalization of John domains. Indeed, we assume inequality of the form

$$|f(x) - f_B| \leq C_* \int_B g(y) \frac{\rho(x,y)}{\mu(B(x,\rho(x,y)))} d\mu(y), \quad f_B = \frac{1}{\mu(B)} \int_B f d\mu,$$

where $x \in B$, $\rho(\cdot,\cdot)$ is a quasimetric, $B(x,\rho(x,y))$ is the quasimetric ball centered at x of radius $\rho(x,y)$, and $g(y)$ is a nonnegative measurable function playing the role of $|\nabla f|$. We use it to deduce

$$|f(x) - f_{Q'}| \leq C \int_{Q_0} g(y) \max\left\{ \frac{\rho(x,y)}{\mu(B(x,\rho(x,y)))}, \frac{\delta\rho(y)}{\mu(B(y,\delta\rho(y)))} \right\} d\mu(y) \\ + \int_{\left(\bigcup_{i=0}^{N(x)} Q_i\right) \setminus Q_0} g(y) \frac{\delta\rho(y)}{\mu(B(y,\delta\rho(y)))} d\mu(y).$$

where $f_{Q'} = \mu(Q')^{-1} \int_{Q'} f d\mu$.

Conde Alonso, José Manuel (Brown University, USA)

Many Calderón-Zygmund operators are not bounded on L^2 with totally irregular measures

Abstract: We consider totally irregular measures μ in $\mathbb{R}^{n+1}$, that is, measures μ such that

$$\limsup_{r\to 0} \frac{\mu(B(x,r))}{(2r)^n} > 0 \ \ \& \ \ \liminf_{r\to 0} \frac{\mu(B(x,r))}{(2r)^n} = 0$$

for μ almost every x. A martingale difference decomposition based on to the so-called David-Mattila cubes allows us to show the following: if

$$T_\mu f(x) = \int K(x,y)\, f(y)\, d\mu(y)$$

is an operator whose kernel $K(\cdot,\cdot)$ is the gradient of the fundamental solution for a uniformly elliptic operator in divergence form associated with a matrix with Hölder continuous coefficients, then T_μ is not bounded on $L^2(\mu)$.

Our result extends a celebrated result by Eiderman, Nazarov and Volberg that holds for the n-dimensional Riesz transforms. It can be viewed as part of the program to clarify the connection between rectifiability of sets/measures on $\mathbb{R}^{n+1}$ and boundedness of singular integrals there.

Based on joint work with Mihalis Mourgoglou and Xavier Tolsa.

Domínguez, Oscar (Universidad Complutense de Madrid, Spain)

Besov spaces of small smoothness: Fourier characterizations and comparisons

Abstract: In this talk we focus on Besov spaces of smoothness close to zero. For $1 \leq p \leq \infty, 0 < q \leq \infty$ and $-\infty < b < \infty$, the Besov space $\mathbb{B}^{0,b}_{p,q}(\mathbb{R}^d)$ is formed by all p-integrable functions f having a finite quasi-norm

$$\|f\|_{\mathbb{B}^{0,b}_{p,q}(\mathbb{R}^d)} = \|f\|_{L_p(\mathbb{R}^d)} + \left(\int_0^1 (1-\log t)^{bq} \omega_k(f,t)_p^q \frac{dt}{t}\right)^{1/q}.$$

Here $k \in \mathbb{N}$ and $\omega_k(f,t)_p$ is the $L_p(\mathbb{R}^d)$-moduli of smoothness of order k. In particular, if $b = 0$ in $\mathbb{B}^{0,b}_{p,q}(\mathbb{R}^d)$ we get the L_p-Dini condition, which has been widely used to solve several questions in harmonic analysis such as boundedness properties of singular integrals and convergence problems of trigonometric series. Even though the spaces $\mathbb{B}^{0,b}_{p,q}(\mathbb{R}^d)$ are close to $L_p(\mathbb{R}^d)$, they have several special properties due to their structure as smoothness spaces, but the lack of classical smoothness yields differents results as compared to the well-understood setting of Besov spaces of positive smoothness. Consequenlty, in recent times there has been a growing interest in understanding better the structure of the spaces $\mathbb{B}^{0,b}_{p,q}(\mathbb{R}^d)$ not only from the point of view of the theory of function spaces, but also in applications to approximation theory, harmonic analysis, probability theory, or applications to PDE's (Bressan's conjecture on mixing flows).

Our goal is two-fold. First, we study the description of the spaces $\mathbb{B}^{0,b}_{p,q}(\mathbb{R}^d)$ in terms of the Fourier transform. Secondly, we give a full answer to a question posed by Andreas Seeger on precise relationships between $\mathbb{B}^{0,b}_{p,q}(\mathbb{R}^d)$, their Fourier-analytically defined counterparts $B^{0,b}_{p,q}(\mathbb{R}^d)$ and the Triebel-Lizorkin spaces $F^{0,b}_{p,q}(\mathbb{R}^d)$. In particular, we prove $\mathbb{B}^{0,b}_{2,2}(\mathbb{R}^d) = B^{0,b+1/2}_{2,2}(\mathbb{R}^d)$. Our approach relies on characterizations of the above mentioned smoothness spaces in terms of the Fourier transform for general monotone functions and lacunary Fourier series.

Based on joint works with F. Cobos (Madrid), S. Tikhonov (Barcelona) and H. Triebel (Jena).

Engelstein, Max (MIT, USA)

A Epiperimetric approach to singular points in the Alt-Caffarelli functional

Abstract: We prove a uniqueness of blowups result for isolated singular points in the free boundary of minimizers to the Alt-Caffarelli functional. The key tool is a (log-)epiperimetric inequality, which we prove by means of two Fourier expansions (one on the function and one on its free boundary).

If time allows we will also explain how this approach can be adapted to (re)prove old and new regularity results for area-minimizing hypersurfaces. This is joint work with Luca Spolaor (Princeton/MIT) and Bozhidar Velichkov (Universite Grenoble Alpes).

Hong, Sunggeum (Chosun University, S. Korea)

Maximal averages over certain non-smooth and non-convex hypersurfaces

Abstract: We consider the maximal operators whose averages are taken over some non-smooth and non-convex hypersurfaces. For each $1 \le i \le d-1$, let $\phi_i : [-1,1] \to \mathbb{R}$ be a continuous function satisfying some derivative conditions, and let $\phi(y) = \sum_{i=1}^{d-1} \phi_i(y_i)$. We prove the L^p boundedness of the maximal operators associated with the graph of ϕ which is a non-smooth and non-convex hypersurface in $\mathbb{R}^d$, $d \ge 3$. This is a joint work with Yaryong Heo and Chan Woo Yang.

Jaye, Ben (Clemson University, USA)

The measures with an associated square function operator bounded in L^2

Abstract: We shall describe an extension of a theorem of David and Semmes ('91) to general non-atomic measures. The result provides a geometric characterization of the non-atomic measures μ for which a certain class of square function operators, or singular integral operators, are bounded in $L^2(\mu)$. The description is given in terms of a modification of Jones' β-coefficients. Joint work with Fedor Nazarov and Xavier Tolsa.

Marín, Juan José (ICMAT, Spain)

Layer potentials and boundary value problems on SKT domains

Abstract: We use the theory of layer potentials to study boundary value problems on SKT domains, a family of domains introduced by S. Semmes, C. Kenig and T. Toro. We consider the case of elliptic systems with constant coefficients in which the boundary data belongs to $L^p(\partial\Omega, w)$, where $w \in A_p(\partial\Omega)$ is a Muckenhoupt weight. This extends previous results by S. Hofmann, M. Mitrea and M. Taylor. This approach relies on the invertibility of layer potentials defined on the boundary of the domain and therefore several tools are developed for this purpose, such as a quantitative interpolation theorem for compact operators. Moreover, these techniques can also be used for the cases of boundary data in other spaces, such as variable Lebesgue spaces or rearrangement-invariant Banach function spaces. This is joint work with J.M. Martell and M. Mitrea.

Merchán, Tomás (Kent State University, United States)

On the relation between L^2 boundedness and existence of principal value integral for a Calderón-Zygmund operator

Abstract: In 1998, Tolsa proved that any measure for which the Cauchy transform operator is bounded in $L^2(\mu)$ also exists in the sense of principal value. However, it turns out that this is not the case in general. Jaye and Nazarov created a measure μ in $\mathbb{C}$ satisfying linear growth for which the singular integral operator with kernel $K(z) = \frac{1}{|z|}\left(\frac{\bar{z}}{|z|}\right)^3$ is bounded in $L^2(\mu)$ but fails to exist in the sense of principal value. In the talk, we will introduce sharp sufficient conditions on a measure μ which ensures that if a Calderón-Zygmund operator is bounded with respect to $L^2(\mu)$, then the operator exists in the sense of principal value. This is a joint work with Benjamin Jaye.

Ocáriz, Jesús (UAM-ICMAT, Spain)

On the topological dual of variable Lebesgue spaces

Abstract: Given an open interval $I \subseteq \mathbb{R}$ and a measurable function $p : I \to [1, +\infty)$, the variable Lebesgue space $L^{p(\cdot)}(I)$ is the subspace of measurable functions $f : I \to \mathbb{R}$ such that the following norm is finite

$$\|f\|_{L^{p(\cdot)}(I)} = \inf\left\{\lambda > 0 : \int_I \left|\frac{f(x)}{\lambda}\right|^{p(x)} \mathrm{d}x \leq 1\right\}.$$

The topological dual of this Banach space is perfectly known when $\|p\|_{L^\infty(I)} < \infty$. However, if $\|p\|_{L^\infty(I)} = \infty$, describing it has been an open problem for years.

In this talk, we are going to discuss some recent approaches that give a better understanding of the phenomena beyond it.

Joint work with A. Amenta (Technische Universiteit Delft), J.M. Conde-Alonso (Brown University) and D. Cruz-Uribe (University of Alabama).

Prisuelos Arribas, Cruz (ICMAT, Spain)

The weighted regularity problem for degenerate elliptic operators

Abstract: Given a weight $w \in A_2$, we consider the following degenerate elliptic operator in $\mathbb{R}^{n+1}_+$, $n \geq 2$,

$$\mathbb{L}_w = -w^{-1}\text{div}_{x,t}\,\mathbb{A}_w\nabla_{x,t}.$$

We show that the weighted regularity problem: $\mathbb{L}_w u = 0$ in $\mathbb{R}^{n+1}_+$ is solvable. That is, for every $f \in \dot{W}^{1,p}(vdw)$ the associated solution with boundary data f given by $u(x,t) := e^{-t\sqrt{L_w}}f(x)$, $(x,t) \in \mathbb{R}^{n+1}_+$, satisfies the following weighted non-tangential maximal function estimate:

$$\|\mathcal{N}_w(\nabla_{x,t}u)\|_{L^p(vdw)} \leq C\|\nabla f\|_{L^p(vdw)}.$$

Our approach is direct and it makes not use of Hardy space theory.

This is a joint work with L. Chen and J.M. Martell

Saari, Olli (University of Bonn, Germany)

Spherical means and derivatives of fractional maximal functions

Abstract: Let χ be the L^1 normalised characteristic function of the unit ball $B(0,1) \subset \mathbb{R}^n$ and let σ be the normalised $n-1$ dimensional surface measure of the unit sphere. Define the fractional maximal functions

$$M_\alpha f = \sup_{r>0} |r^\alpha \chi_r * f|, \quad S_\alpha f = \sup_{r>0} |r^\alpha \sigma_r * f|,$$

where $f \in L^2(\mathbb{R}^n)$ and $\alpha \in (0,n)$. It is well known that the M_α is comparable to the Riesz potential I_α at the level of L^p norms. Moreover, it was proved by Kinnunen and Saksman that $\nabla M_\alpha f$ exists as a locally integrable function whenever $f \in L^p$ with $p > 1$ and $\alpha \in [1,n)$. I will discuss an analogous result concerning the derivatives of $S_\alpha f$ in dimensions $n \geq 5$. This is joint work with David Beltran and João P. Ramos.

Turner, Andrew (University of Birmingham, UK)

Solvability of boundary value problems for the Schrödinger equation with non-negative potentials

Abstract: We consider Schrödinger equations of the form $\operatorname{div} A\nabla u + Vu = 0$, where V is a non-negative potential and A is a complex-valued, bounded multiplication matrix. We prove well posedness for boundary value problems in the upper half-space with L^2-Neumann and Dirichlet boundary data when A is complex Hermitian or of a certain block-type. We build on methods for unperturbed second-order elliptic systems by incorporating the potential V as an additive perturbation of a first order system DB with bounded holomorphic functional calculus.