EXCELENCIA SEVERO OCHOA

ICMAT
INSTITUTO DE CIENCIAS MATEMÁTICAS

Real Harmonic Analysis and its Applications to Partial Differential Equations and Geometric Measure Theory

On the occasion of the 60th birthday of Steve Hofmann

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Abstracts Main Talks

Auscher, Pascal (Centre National de la Recherche Scientifique, LAMFA, France)

Existence and uniqueness for non-autonomous parabolic Cauchy problems with rough coefficients

Abstract: I discuss the initial value problem for parabolic equations or systems in the form $\partial_t u = \mathrm{div} A\nabla u$ on the upper half space, with initial data in L^p spaces. After giving a construction of propagators through an optimal energy space, I present a criterion that allows one to obtain interior représentation of solutions and is the key to trace and uniqueness results under weak interior control. In particular, no control when $t \to \infty$ is required to obtain uniqueness for real equations with L^p data, which seems not to have been observed. This is based on a joint work with S. Monniaux et P. Portal.

Barton, Ariel (University of Arkansas, USA)

The Neumann problem for symmetric higher order elliptic differential equations

Abstract: Second-order equations of the form $\nabla \cdot A\nabla u = 0$, with A a uniformly elliptic matrix, have many applications and have been studied extensively. A well-known foundational result of the theory is that, if the coefficients A are real-valued, symmetric, and constant along the vertical coordinate (and merely bounded measurable in the horizontal coordinates), then the Dirichlet problem with boundary data in L^2 or $\dot{W}_1^2$ and the Neumann problem with boundary data in L^2 are well-posed in the upper half-space.

The theory of higher-order elliptic equations of the form $\nabla^m \cdot A\nabla^m u = 0$ is far less well understood. In this talk we will establish well posedness of the L^2 Neumann problem in the half-space, for higher-order equations with real symmetric vertically constant coefficients; this improves on our earlier work by discussing nontangential as well as square function estimates.

David, Guy (Université de Paris Sud, France)

Harmonic measure with lower dimensional boundaries

Abstract: This will describe joint work with Max Engelstein, Joseph Feneuil, and Svitlana Mayboroda. Let Γ be an Ahlfors-regular set of dimension $d < n-1$ in $\mathbb{R}^n$, and set $\Omega = \mathbb{R}^n \setminus \Gamma$ (a nice, well connected domain). We define a reasonable notion of harmonic measure on $\Gamma = \partial\Omega$, still defined in terms of a locally elliptic differential operator of order 2, but which is degenerate at the boundary and depends on Γ. We study the mutual absolute continuity of this harmonic measure and d-dimensional Hausdorff on Γ, in terms of geometric regularity properties of Γ such as (hopefully) uniform rectifiability.

Iosevich, Alex (University of Rochester, USA)

Fuglede, Riesz and Gabor: analytic, combinatorial and number theoretic aspects of the existence problems for exponential bases

Abstract: Let Ω be a bounded domain in $\mathbb{R}^d$. We say that Ω is Fuglede spectral if $L^2(\Omega)$ possesses an orthogonal bases of exponentials $\{e^{2\pi ix\cdot a}\}_{a\in A}$. We say that it is Riesz spectral if $L^2(\Omega)$ possesses a Riesz basis of the same form. Finally, we say that χ_Ω is a window function for an orthogonal Gabor system if there exists $S \subset \mathbb{R}^{2d}$ such that $\{e^{2\pi ix\cdot a}\chi_\Omega(x-b)\}_{(a,b)\in S}$ is an orthogonal basis of $L^2(\mathbb{R}^d)$.

We are going to discuss the history of these problems and connections between them, with the focus on a variety of techniques from different areas of mathematics that inevitably come up. We will also establish some recently proved results about the existence and non-existence of Gabor bases in a variety of settings. Connections with combinatorial geometry and multi-dimensional harmonic analysis play a key role.

Jerison, David (MIT, USA)

The Two Hyperplane Conjecture

Abstract: I will introduce a conjecture that I call the *Two Hyperplane Conjecture*, saying that an isoperimetric surface that divides a convex body in half by volume is trapped between parallel hyperplanes. Emanuel Milman has shown that in its strongest, dimension-independent form, my conjecture implies the *Hyperplane Conjecture* of Kannan, Lovász and Simonovits in theoretical computer science, which says that the area of such an isoperimetric surface is comparable, by an absolute constant independent of dimension and the convex body, to the area of some hyperplane dividing the convex body in half. Their conjecture is closely related to several famous unsolved problems in high dimensional convex geometry. But unlike the hyperplane conjecture, the two-hyperplane conjecture has significance even in low dimensions.

I will relate the conjecture to qualitative and quantitative connectivity properties and regularity of area-minimizing surfaces, free boundaries and level sets of eigenfunctions, and report on work in progress with Guy David. The main theme of the talk is that the level sets of least energy solutions to scalar variational problems should be as simple as possible, but no simpler.

Kenig, Carlos (University of Chicago, USA)

The method of channels of energy

Abstract: We will discuss a new method, introduced in joint works with Duyckaerts and Merle, which allows us to study the long-time behavior of large solutions to nonlinear wave equations, and in particular to address the issue of soliton resolution for them.

Lacey, Michael T. (Georgia Institute of Technology, USA)

A Surplus of Sparse Bounds

Abstract: The sparse bound is the domination of a bilinear form in terms of a simple sum of local averages of the functions in the form. This bound represents a particular quantification of the weak-type bounds. Once established, immediately gives a wide range of weighted inequalities, for weights in the intersection of Muckenhoupt and Reverse Holder classes. These bounds are fully quantitative. The sparse bound applies to lots of different operators. We will survey these results.

Lewis, John (University of Kentucky, USA)

Existence and Regularity of a Capacitary Functions in a Minkowski Inspired Geometric Problem

Abstract: In the first part of this talk, given K a compact convex set with nonempty interior and u the A capacitary function of the complement of K, we introduce a measure μ_K (corresponding to u, A) with support in the unit sphere (denoted $\mathbf{S}^{n-1}$) of Euclidean n space = $\mathbf{R}^n$. Here $\nabla \cdot A(\nabla u) = 0$ weakly in the complement of K where $A : \mathbf{R}^n \setminus \{0\} \to \mathbf{R}^n$ is homogeneous of degree $p-1$ and satisfies p Laplacian type structure condtions for fixed $p, 1 < p < n$. Second given a finite positive Borel measure ν, defined on $\mathbf{S}^{n-1}$, with the property that (a) the support of ν is not contained in any great circle and (b) the centroid of ν is at the origin, we outline a proof that shows for certain A, there always exists K with $\nu = \mu_K$. Third, if $d\nu = gd\sigma$ where σ is surface area on $\mathbf{S}^{n-1}$, we discuss what regularity of g implies about regularity of ∂K when $\nu = \mu_K$.

Mitrea, Dorina (University of Missouri, USA)

The Poisson integral representation formula for elliptic systems in rough domains

Abstract: In this talk I will present an integral representation formula for solutions of the Dirichlet problem for a given weakly elliptic second-order system L, in domains of a very general geometric nature, which involves the conormal derivative of the Green function for the transposed system $L^\top$ as integral kernel.

Mitrea, Marius (University of Missouri, USA)

Fatou Theorems for Elliptic Systems in Uniformly Rectifiable Domains

Abstract: The trademark blueprint of a Fatou-type theorem is that size and integrability properties of the nontangential maximal operator for a null-solution of an elliptic equation in a certain domain implies the a.e. existence of the pointwise nontangential boundary trace of the said function. It is natural to call such a theorem quantitative if the boundary trace does not just simply exists but encodes significant information regarding the size of the original function.

In this talk, I will be presenting a quantitative Fatou-type theorem for null-solutions of an injectively elliptic first-order (homogeneous, constant complex coefficient) system of differential operators in an arbitrary uniformly rectifiable domain in the n-dimensional Euclidean space, assuming that the nontangential maximal operator is p-th power integrable (with respect to the Hausdorff measure) for some integrability exponent larger than $(n-1)/n$. Such a result has a wide range of applications, including the theory of Hardy spaces associated with injectively elliptic first-order systems in uniformly rectifiable domains. The approach I develop, also produces a Fatou-type theorem for the gradient of null-solutions of elliptic second-order systems in arbitrary uniformly rectifiable domains.

Nyström, Kaj (Uppsala University, Sweden)

The Dirichlet problem for second order parabolic operators in divergence form

Abstract: I will discuss the L^p Dirichlet problem for parabolic equations with real, time-dependent, elliptic coefficients.

Pipher, Jill (Brown University, USA)

Boundary value problems for complex coefficient p-elliptic operators

Abstract: This lecture will report on joint work with Martin Dindos: we consider Dirichlet and regularity problems, as well as perturbation theory, for a class of elliptic operators with complex coefficients satisfying the p-ellipticity condition. Our assumption is a strengthening of a condition introduced by Cialdea and Maz'ya in connection with their work on L^p-dissipativity, and was also investigated by Carbonaro and Dragicevic in their work on boundedness of bilinear forms, where the term p-elliptic originates.

Sawyer, Eric (McMaster University, Canada)
Two weight Tb theorems and energy conditions

Abstract: We discuss recent work on two weight Tb theorems and energy conditions, including joint work with Ignacio Uriarte-Tuero and Chun-Yen Shen.

Tolsa, Xavier (ICREA and Universitat Autònoma de Barcelona, Spain)
Favard length, analytic capacity, and the Cauchy transform

Abstract: In 1960's Vitushkin conjectured that a compact set in the plane is non-removable for bounded analytic functions (or equivalently, has positive analytic capacity) if and only it has positive Favard length, or in other words, its orthogonal projections have positive length in a set of directions of positive measure. In 1986 Mattila showed that this conjecture is false. However, it is not known yet if one of the implications in Vitushkin's conjecture holds. Namely, does positive Favard length imply positive analytic capacity? In this talk I will present a joint result with Alan Chang related to this open question. In a sense, this asserts that if one strengthens the assumption of positive Favard length in a suitable way, then the answer is positive. More precisely, if the density of a projected measure is in L^2 in an interval of directions, then the Cauchy transform with respect to this measure is bounded in L^2 and analytic capacity is positive.

Toro, Tatiana (University of Washington, USA)
Elliptic measure and rectifiability

Abstract: In this talk we will describe some recent results concerning the relationship between the behavior of the elliptic measure for certain divergence form elliptic operators and the geometry of the boundary of the domain where the operators are defined. The results bear a strong resemblance to those obtained for the harmonic measure. This is joint work with S. Hofmann, J.M. Martell, S. Mayboroda and Z. Zhao.

Volberg, Alexander (Michigan State University, USA)
Using tree to climb on hypercube

Abstract: We will explain a new unified approach to classical isoperimetric and geometric functional inequalities with Gaussian measure and on Hamming cube. The method involves solving a certain Monge-Ampère equation by the exterior differential systems approach of Bryant-Griffiths. Then we explain that underlying this new method there is a series of inequalities on Hamming cube which are considerably more difficult to obtain. In some situation we show that we can find "the Bellman function" of the problem or rather its Legendre transform that solves the corresponding extremal problem on Hamming cube. This Bellman function lives in a totally different world: the world of harmonic analysis on trees, or martingale harmonic analysis. Why tis duality reveals itself in so many ways, we do not know, but the examples of pairs (Bellman function for martingale (tree), classical geometric inequality on Hamming cube) include 1) (Chang–Wilson-Wolff function, log-Sobolev inequality), 2) (Bollobàs function, isoperimetric inequality on Hamming cube), 3) (Burkholder–Gundy and Davis functions, improved Beckner's inequalities on Hamming cube). Extremal problems on Hamming cubes are closely related to "big data" paradigm, so one can consider this talk (based on papers joint with P. Ivanisvili and F. Nazarov) as our attempt to become an applied mathematicians.