Volberg, Alexander (Michigan State University, USA) Using tree to climb on hypercube

Abstract: We will explain a new unified approach to classical isoperimetric and geometric functional inequalities with Gaussian measure and on Hamming cube. The method involves solving a certain Monge-Ampère equation by the exterior differential systems approach of Bryant-Griffiths. Then we explain that underlying this new method there is a series of inequalities on Hamming cube which are considerably more difficult to obtain. In some situation we show that we can find "the Bellman function" of the problem or rather its Legendre transform that solves the corresponding extremal problem on Hamming cube. This Bellman function lives in a totally different world: the world of harmonic analysis on trees, or martingale harmonic analysis. Why tis duality reveals itself in so many ways, we do not know, but the examples of pairs (Bellman function for martingale (tree), classical geometric inequality on Hamming cube) include 1) (Chang–Wilson-Wolff function, log-Sobolev inequality), 2) (Bollobàs function, isoperimetric inequality) on Hamming cube), 3) (Burkholder–Gundy and Davis functions, improved Beckner's inequalities on Hamming cube). Extremal problems on Hamming cubes are closely related to "big data" paradigm, so one can consider this talk (based on papers joint with P. Ivanisvili and F. Nazarov) as our attempt to become an applied mathematicians.