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Favard length, analytic capacity, and the Cauchy transform

Abstract: In 1960's Vitushkin conjectured that a compact set in the plane is non-removable for bounded analytic functions (or equivalently, has positive analytic capacity) if and only if it has positive Favard length, or in other words, its orthogonal projections have positive length in a set of directions of positive measure. In 1986 Mattila showed that this conjecture is false. However, it is not known yet if one of the implications in Vitushkin's conjecture holds. Namely, does positive Favard length imply positive analytic capacity? In this talk I will present a joint result with Alan Chang related to this open question. In a sense, this asserts that if one strengthens the assumption of positive Favard length in a suitable way, then the answer is positive. More precisely, if the density of a projected measure is in L^2 in an interval of directions, then the Cauchy transform with respect to this measure is bounded in L^2 and analytic capacity is positive.