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Fuglede, Riesz and Gabor: analytic, combinatorial and number theoretic aspects of the existence problems for exponential bases

Abstract: Let Ω be a bounded domain in $\mathbb{R}^d$. We say that Ω is Fuglede spectral if $L^2(\Omega)$ possesses an orthogonal bases of exponentials $\{e^{2\pi i x\cdot a}\}_{a\in A}$. We say that it is Riesz spectral if $L^2(\Omega)$ possesses a Riesz basis of the same form. Finally, we say that χ_Ω is a window function for an orthogonal Gabor system if there exists $S \subset \mathbb{R}^{2d}$ such that $\{e^{2\pi i x\cdot a}\chi_\Omega(x-b)\}_{(a,b)\in S}$ is an orthogonal basis of $L^2(\mathbb{R}^d)$.

We are going to discuss the history of these problems and connections between them, with the focus on a variety of techniques from different areas of mathematics that inevitably come up. We will also establish some recently proved results about the existence and non-existence of Gabor bases in a variety of settings. Connections with combinatorial geometry and multidimensional harmonic analysis play a key role.