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Supercaloric functions for the porous medium equation

Abstract: We consider a class of nonnegative supersolutions to the porous medium equation

$$u_t - \Delta(u^m) = 0,$$

in the slow diffusion case $m > 1$. These supersolutions are defined as lower semicontinuous functions obeying comparison principle and they are called m-supercaloric functions. For the ordinary heat equation we have supercaloric functions.

The leading example of a m-supercaloric function with a point singularity is the Barenblatt solution, which corresponds to the fundamental solution for the heat equation. In the slow diffusion case so-called friendly giant and other examples constructed by separation of variables are included in the theory. Their infinity sets occupy a whole time slice and they may blow up arbitrarily fast near the polar set.

This talk reviews the definition and properties of m-supercaloric functions. Connections to weak supersolutions, measure data problems and the Perron–Wiener–Brelot method are also explained. We show there are two mutually exclusive alternatives: every nonnegative m-supercaloric function has either a Barenblatt type behavior or blows up at least with the rate given by the friendly giant. Several characterizations for both cases are given.