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Diffusion and wave propagation next to irregular boundaries: harmonic measure, wave absorption, and noise abatement wall

Abstract: The diffusion equation in a bounded domain with a partially absorbing boundary reads

$$-D\Delta u + \frac{\partial u}{\partial t} = 0 \quad , \quad \left(\frac{\partial u}{\partial n} + \frac{u}{\Lambda}\right)\Big|_{\partial\Omega} = 0$$

where Λ is a characteristic length which depends on the permeability of the boundary. Acoustic waves in the same domain Ω solve the wave equation with similar boundary condition

$$-\Delta u + \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = 0 \quad , \quad \left(\frac{\partial u}{\partial n} + \frac{u}{\Lambda}\right)\Big|_{\partial\Omega} = 0$$

where Λ now depends on the absorption at the boundary. In both cases, the evolution of an initial-value problem is entirely determined from the knowledge of the eigenfunctions of the Laplace operator in the domain. We will study the role played by a geometrical irregularity or complexity of the boundary and establish a connection between the harmonic measure of the boundary and its performance for absorbing waves. We will show in particular that in the high absorption or high permeability regime, large gradients of the solution are observed at the boundary. This property can be used to design very efficient noise abatement structures.