Azzam, Jonas (University of Edinburgh, UK)

Wasserstein distance and rectifiability of measures

Abstract: A curve of finite length has tangents at almost every point (with respect to arclength), which follows from arclength parametrizing the curve and applying Rademacher's theorem. Thus, as we zoom in at almost every point on the curve, the curve is getting flatter and flatter. However, asymptotic flatness at every point is not sufficient for a curve to be rectifiable. A result of Bishop and Jones, however, shows that a necessary and sufficient condition for rectifiability is that how straight the curve is in a ball of radius r is square integrable over the radii. This is analogous to results in harmonic analysis that classify the pointwise differentiability of a function in terms of a Dini condition. In this talk, we will discuss a variant of this result for Radon measures in Euclidean space and show that, for pointwise doubling measures, rectifiability of a measure is equivalent to a similar Dini condition that measures "flatness" in terms of how close (using a variant of Wasserstein distance) the measure is to resembling planar measure. This is joint work with Xavier Tolsa and Tatiana Toro.