New Trends in Harmonic Analysis at ICMAT

Workshop on New Trends on Noncommutative Harmonic Analysis
December 17th-21th, 2012

Aula Gris 1 (ICMat, Campus de Cantoblanco)

Monday 17  12:15-13:30  Marius Junge
Monday 17  15:45-17:00  Steve Avsec
Tuesday 18  10:30-11:45  Uwe Franz
Tuesday 18  12:15-13:30  Tao Mei
Wednesday 19  10:30-11:45  Éric Ricard
Wednesday 19  12:15-13:30  Mathilde Perrin
Wednesday 19  15:45-17:00  Mikael de la Salle
Thursday 20  10:30-11:45  Stephan Neuwirth
Thursday 20  12:15-13:30  Yanqi Qiu
Friday 21  10:30-11:45  Quanhua Xu

Abstracts:

What means Noncommutative Harmonic Analysis here?
Marius Junge, University of Illinois at Urbana-Champaign

In this talk we discuss what noncommutative harmonic analysis on noncommutative spaces is not, and what it could be. We will discuss some results from joint work with Mei and Parcet, and try to explore where the connection between geometry and Calderon-Zygmund operators plays a role. Certainly, we are just at the beginning of a longer journey.
A CLASSIFICATION OF EXCHANGEABLE NONCOMMUTATIVE BROWNIAN MOTIONS
Steve Avsec, Université de Franche-Comté

In this talk, we will first discuss a classification of exchangeable noncommutative brownian motions (following the recent definition of Collins and Junge) in terms of certain positive definite functions on the infinite symmetric group. We will then discuss a number of examples which suggest that this correspondence may be many-to-one. Possibilities for a complete characterization in terms of combinatorial data and other possibilities for future work will also be discussed.

HEAT SEMIGROUPS ON COMPACT QUANTUM GROUPS
Uwe Franz, Université de Franche-Comté

We study candidates for “heat semigroups” on compact quantum groups. Our examples include $S_N^+, O_N^+$, and $SU_q(2)$. Joint work with Fabio Cipriani and Anna Kula.

BMO AND MEASURABLE OPERATORS
Tao Mei, Wayne State University

Let $(\mathcal{M}, \tau)$ be a semifinite von Neuman algebra with a n.f.s. trace. Let $(T_t), 0 < t < \infty$, be a Markov semigroup of u.c.p maps on $\mathcal{M}$. Recent joint work with Junge and Parcet relies on a type of noncommutative BMO spaces associated with $T_t$, an analogue of Pisier/Xu’s BMO space for noncommutative martingales. For $x \in L_2(\mathcal{M})$, let

$$\|x\|_{bmo_c} = \sup_t \|T_t|x|^2 - |T_t x|^2\|^\frac{1}{2}.$$ 

Note the linear space of all $x \in L_2(\mathcal{M})$ with a finite $bmo_c$ norm may not be complete with respect to the $bmo_c$-norm, unless $\mathcal{M}$ is finite. In our recent articles, we complete the linear space in an abstract way so that the abstract completion, called $bmo_c$ space, to have the desired properties.

It would be nice to identify a linear space associated with $\mathcal{M}$ and $T_t$, call it “$T$-integrable” operators, such that the collection of all “$T$-integrable” operators with finite $bmo_c$-norm is complete automatically. In this talk, I will try to include necessary background information and try to inspire a discussion on related questions.

PERTURBATIONS OF OPERATOR ALGEBRAS
Éric Ricard, Université de Caen

We try to analyze small deformations of operator algebras. We present an operator space version of the Amir-Cambern theorem and discuss related open questions.
HYPERCONTRACTIVITY FOR SOME FOURIER MULTIPLIERS
Mathilde Perrin, ICMAT

Hypercontractivity is a widely studied topic in many different situations, mostly from a probabilistic point of view. In this talk, we propose to look at the trigonometric point of view, by studying the hypercontractive property for semigroups which are Fourier multipliers. More precisely, we are interested in “Poisson-like” semigroups acting on von Neumann algebras associated to discrete groups. By a combinatorial approach, hypercontractive estimates are obtained for a large class of groups. However, these estimates are not optimal. We will discuss a method, combining combinatorial arguments with a numerical part, which can yield optimal hypercontractive inequalities $L_2 \rightarrow L_q$ (with $q$ even integer) for groups having some nice properties. In particular, we obtain optimal results for $F_2$, for some cyclic groups $\mathbb{Z}_n$, and for some triangle groups $\Delta(\ell, m, n)$.

SOME QUESTIONS ON THE RELATION BETWEEN FOURIER AND SCHUR MULTIPLIERS
Mikael de la Salle, Université de Franche-Comté

To a function $\varphi$ on a group $G$, one can associate a Schur multiplier, which maps a matrix $(a_{s,t})_{s,t \in G}$ to the matrix $(\varphi(s^{-1}t)a_{s,t})_{s,t \in G}$, and a Fourier multiplier, which maps a function $f$ on $G$ to the function $\varphi f$. It is a well-known and classical result from Bozejko and Fendler that the completely bounded norms of such a Schur multiplier on $B(L^2(G))$ and of the Fourier multiplier on $VN(G)$ coincide. The first question I will address is: what remains of this for non-commutative $L^p$ spaces? I will discuss some partial positive results in the amenable case, generalizing a theorem of Neuwirth and Ricard to the locally compact case (joint with M. Caspers), as well as other questions and negative results. The motivation is the question whether, for a discrete group $\Gamma$, the property $AP_{Schur}^{\text{pcb}}$ that I introduced with V. Lafforgue is an invariant of the von Neumann algebra of $\Gamma$.

LACUNARY SETS IN DISCRETE GROUPS AND LACUNARY MATRICES
Stefan Neuwirth, Université de Franche-Comté

I shall try to give a survey on results and problems on lacunary sets in discrete groups and lacunary matrices, with an emphasis on commutative results that still do not have their noncommutative counterpart.

COMPLETELY BOUNDED PALEY PROJECTIONS ON SOBOLEV SPACES ON TORI
Yanqi Qiu, Université Paris 6

A necessary and sufficient condition on the smoothness under which there exist Paley projection on the Sobolev spaces on tori was characterized by Pelczynski and Wojciechowski. We obtain the complete boundedness of the Paley projections considered by them. Potential applications will also be mentioned.

A NONCOMMUTATIVE HELSON-SZEGO THEOREM WITH APPLICATIONS TO TOEPLITZ OPERATORS
Quanhua Xu, Université de Franche-Comté

We present a Helson-Szego theorem for subdiagonal algebras (in particular, for algebras of matrix-valued analytic functions in the unit disc). We then apply this result to characterize the invertibility of Toeplitz operators on the associated Hardy spaces.