On a problem of David and Semmes in a co-dimension one case
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Tuesday 7 10:00 - 12:00  Instituto de Ciencias Matemáticas
Wednesday 8 10:00 - 12:00  Campus de Cantoblanco
Thursday 9 10:00 - 12:00  Aula Naranja
Friday 10 10:00 - 12:00

Abstract:

The boundedness of the Riesz operator (whose kernel is the gradient of the fundamental solution for Laplacian in $\mathbb{R}^d$) in $L^2$ with respect to $d - 1$ dimensional Hausdorff measure must imply the rectifiability of this measure. This statement became known as David–Semmes problem. It has been proved only for $d = 2$, first by Mattila–Melnikov–Verdera for the case of homogeneous set, and later by Tolsa in a non-homogeneous situation. The non-homogeneous situation for $d = 2$ also involves relations between beta numbers of Peter Jones and Menger’s curvature. The proof in both cases involves the so-called Menger’s curvature, which is “cruelly missing” (by the expression of Guy David) in dimensions $d > 2$. In a recent work of Nazarov–Tolsa–Volberg the conjecture of David and Semmes has been validated. The proof (which does not involve Menger’s curvature) gives a new and much different proofs of the abovementioned results also in the case $d = 2$. The result can be cast in the language of the existence of bounded harmonic vector fields in certain (infinitely connected) domains. In fact our result is a certain co-dimension 1 claim. In higher co-dimensions the problem rests open.