Harmonic Measure and Uniform Rectifiability
May 16th-22nd, 2013

Steve Hofmann  University of Missouri

Thursday 16  11:30 - 13:30  Instituto de Ciencias Matemáticas
Friday 17    11:30 - 13:30  Campus de Cantoblanco
Tuesday 21   10:00 - 12:00  Aula Naranja
Wednesday 22 15:00 - 17:00

Abstract: Joint work with J.M. Martell.

We present a higher dimensional, scale-invariant version of the classical theorem of F. and M. Riesz, which established absolute continuity of harmonic measure with respect to arc length measure, for a simply connected domain in the complex plane with a rectifiable boundary. More precisely, for $d \geq 3$, we obtain scale invariant absolute continuity of harmonic measure with respect to surface measure, along with higher integrability of the Poisson kernel, for a domain $\Omega \subset \mathbb{R}^d$, with a uniformly rectifiable boundary, which satisfies the Harnack Chain condition plus an interior (but not exterior) corkscrew condition.

The following is a brief outline of topics to be covered.

I. Background material.
   - Perron’s method/ construction of harmonic measure and the Green function.
   - Bourgain’s estimate: accretivity of harmonic measure.
   - Hölder continuity at the boundary.
   - Comparison principle and “change of pole” formula.
   - Brief review of Uniform Rectifiability

II. Proof of the main theorem.
   - Harnack Chains imply a Poincaré inequality.
   - Dyadic version of the Dahlberg-Jerison-Kenig “sawtooth lemma”.
   - A discrete “Corona decomposition”.
   - Weak reverse Hölder estimates à la Bennewitz-Lewis.
   - “Extrapolation” of Carleson measures.

We remark that there is also a converse, obtained in joint work with J. M. Martell and I. Uriarte-Tuero, in which we deduce uniform rectifiability of the boundary, assuming scale invariant $L^p$ bounds, with $p > 1$, for the Poisson kernel. Time constraints will probably preclude a detailed discussion of the latter result.