Geometric Integrators for non-autonomous dynamical systems

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The numerical integration of differential equations has experienced an important progress during the las few decades. In particular, the *Geometric Integration* (GI), a new branch in Applied Mathematics, as emerged. The evolution of many physical systems is modeled by differential equations which retain the observed qualitative properties of the systems. In general, standard numerical integrators do not preserve these qualitative properties producing, in many cases, wrong qualitative results or leading to inefficient algorithms. It has been widely recognized that the class of numerical integrators which preserve the geometric properties of the exact flow provide a better description of the system.

In this talk we consider GIs for the non-autonomous equation

$$x' = f(x,t), \qquad x(t_0) = x_0 \in \mathbb{R}^d. \tag{1}$$

A usual procedure which enormously simplifies the analysis to search for numerical methods is to transform (1) to autonomous form by appending t to the depending variables

$$\left\{ \begin{array}{rcl} x' & = & f(x,x_t) \\ x_t' & = & 1 \end{array} \right. \tag{2}$$

The system is now autonomous, but it usually presents some drawbacks when applied with geometric integrators which have been tailored for problems with particular structures. Formulation (2) introduces the auxiliary variable x_t aiming to eliminate the explicit time dependence but, the non-autonomous problem frequently looses its original algebraic structure and, in this framework, many highly efficient GIs, suitably designed for autonomous systems, can reveal a significant loss of accuracy. In order to avoid this drawback, in this talk we present several procedures based on splitting techniques and/or on Magnus series which allow, in many relevant cases, to use the highly efficient methods tailored for different classes of non-autonomous dynamical systems.