



Lecture series on

## ***Physical applications of Noncommutative Geometry***

by **Prof. Fedele LIZZI**

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### **PART 1**



#### **UPC - Barcelona**

**September 20 - 22, 2011**

Universitat Politècnica de Catalunya  
Facultat de Matemàtiques i Estadística  
Aula S03

Sept. 20+21, 16:00 – 17:30

Sept. 22, 16:00 – 18:00

#### ***Spectral Geometry and A. Connes' approach to the Standard Model***

Complementary talks:

- Sept. 20, 2011, 17:45 - 18:45  
J. Plazas (Granada)

- Sept. 21, 2011, 17:45 - 18:45  
K. Ebrahimi-Fard (ICMAT, Madrid)

- Sept. 22, 2011, 18:15 - 19:15  
I. Galvez (UAB, Barcelona)

### **PART 2**



#### **ICMAT - Madrid**

**September 27 - 29, 2011**

Instituto de Ciencias Matemáticas  
Sala Gris 1

Sept. 27+28+29, 11:00 – 13:00

#### ***Field Theory on Noncommutative Spaces***

Complementary talks:

- Sept. 27, 2011, 14:30 - 15:30  
J. Varilly (Zaragoza)

- Sept. 28, 2011, 14:30 - 15:30  
F. Lledo (UCIII, Madrid)

- Sept. 29, 2011, 14:30 - 15:30  
F. Ruiz Ruiz (UCM, Madrid)

Organized by

Kurusch Ebrahimi-Fard (ICMAT, Madrid)

Miguel Rodriguez-Olmos (UPC, Barcelona)

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## Aula S03

<b>Tuesday 20/09</b>	<b>Wednesday 21/09</b>	<b>Thursday 22/09</b>
F. Lizzi 16:00-17:30	F. Lizzi 16:00-17:30	F. Lizzi 16:00-18:00
J. Plazas 17:45-18:45	K. Ebrahimi-Fard 17:45-18:45	I. Galvez 18:15-19:15

F. LIZZI lecture

### **Part I: Spectral Geometry and A. Connes' approach to the Standard Model**

I will introduce Connes approach to the description of noncommutative spaces and their geometric properties. This is done by giving an intrinsic algebraic description of ordinary geometry in terms of commutative  $C^*$ -algebras, and then generalize it to the noncommutative case. I will describe the metric properties of these spaces via the generalized Dirac operator, and then apply these ideas to the spectral action to describe the standard model of fundamental particle interactions coupled with gravity. I will take a renormalization approach and discuss anomalies. Time permitting I will discuss other noncommutative spaces like the noncommutative torus or the fuzzy sphere and disc.

- 1.1 Intro and commutative geometry. Gelfand Naimark, GNS.
- 1.2 Spectral Geometry. Dirac Operator and metric properties.
- 1.3 Spectral action.
- 1.4 Standard model.
- 1.5 Noncommutative Torus and Fuzzy spaces.

**Tuesday, Sept. 20, 17:45-18:45**

Jorge PLAZAS (Univ. De Granada)

**Title: From Quantum Statistical Mechanics to Number Theory: The Arithmetic of Noncommutative Spaces**

Abstract: Noncommutative geometry provides a natural framework for various unexpected interactions between physics and number theory, which have arisen in recent years. In this talk we explore the role and relevance of quantum statistical mechanical systems of arithmetic nature. After introducing the basics of this formalism we will discuss some of the central examples (Bost-Connes and Connes-Marcolli systems). Potential applications of this framework and work in progress towards some of these applications will also be discussed.

**Wednesday, Sept. 21, 17:45-18:45**

Kurusch EBRAHIMI-FARD (ICMAT-CSIC, Madrid)

**Title: Feynman graphs, Green functions and related algebraic structures**

Abstract: Based on Dirk Kreimer's seminal findings, Alain Connes and Kreimer developed an algebraic setting encoding essential combinatorial and algebraic aspects underlying the so-called BPHZ renormalization method in perturbative quantum field theory. Key notions are pre-Lie and Hopf algebraic structures of Feynman graphs, and hence Green functions. In this talk we will review these structures, i.e. Connes-Kreimer Hopf algebra of Feynman graphs. Also, we will briefly report on recent joint work with F. Patras (CNRS, Univ. of Nice, France) on the so-called exponential renormalization method. Using Dyson's identity for Green's functions as well as the link between the Faà di Bruno Hopf algebra and Hopf algebras of Feynman graphs, its relation to the composition of formal power series is analyzed. Eventually we analyze the role of the Rota-Baxter property for renormalization scheme maps.

**Thursday, Sept. 22, 18:15-19:15**

Imma GALVEZ (Univ. Auton. De Barcelona)

**Title: Groupoids, and Faà di Bruno formulae for Green functions**

Abstract: The Connes-Kreimer Hopf algebra of trees (or of Feynman graphs) encodes the combinatorics of the BPHZ renormalisation procedure in pQFT. The comultiplication of a tree returns all the ways of "cutting" the tree. However, the individual trees (or graphs) do not have direct physical interpretation; rather certain infinite sums, the so-called Green functions, carry the physical meaning. Van Suijlekom recently discovered that the Green functions satisfy a version of the classical Faà di Bruno formula for substitution of power series. In this talk we will show how the theory of groupoids can be used to give a very conceptual proof of the Faà di Bruno formulae for Green functions in the bialgebra of trees. In this framework a Green function is (the cardinality of) a groupoid and the Faà di Bruno formula is shown to be essentially an equivalence of groupoids (joint work with J Kock, A Tonks).

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## Sala Gris 2

<b>Tuesday 27/09</b>	<b>Wednesday 28/09</b>	<b>Thursday 29/09</b>
F. Lizzi 11:00-13:00	F. Lizzi 11:00-13:00	F. Lizzi 11:00-13:00
J. Varilly 14:30-15:30	F. Lledo 14:30-15:30	F. Ruiz Ruiz 14:30-15:30

F. LIZZI lecture

### Part II: Field theory on Noncommutative Spaces

I will introduce the deformed noncommutative products via the standard example of the quantum phase space. I will then describe field theories on noncommutative spaces based on these products (Moyal and others). I also discuss the (quantum) symmetries of these theories

- 2.1 Moyal product, deformation quantization. Need for noncommutative spacetime.
- 2.2 Scalar fields, Ultraviolet/Infrared mixing.
- 2.3 Alternative products.
- 2.4 Gauge theories.
- 2.5 Twisted symmetries.

**Tuesday, Sept. 27, 14:30-15:30**

Joseph VARILLY (Univ. de Zaragoza and Univ. de Costa Rica, San Jose)

**Title: Riemannian manifolds in the noncommutative way**

Spectral triples with Dirac-like operators can be called noncommutative  $\text{spin}^c$  manifolds. Absent a  $\text{spin}^c$  structure, we can define a noncommutative version of a Riemannian spectral triple. Both kinds of noncommutative manifolds are related using products of unbounded Kasparov modules. The classical correspondence between  $\text{spin}^c$  manifolds and Riemannian manifolds with a  $\text{spin}^c$  structure extends precisely to the noncommutative context, using a bit of KK-theory.

**Wednesday, Sept. 28, 14:30-15:30**

Fernando LLEDO (Univ. Carlos III de Madrid)

**Title: Modular Theory for von Neumann algebras and some applications to quantum physics**

Abstract: In the first part of this talk I will review (as elementary as possible) the Modular Theory for von Neumann algebras and introduce the so-called modular conjugation and modular group. I will illustrate these objects with several elementary examples. In the second part of the talk I will mention several applications of this theory to quantum physics.

**Thursday, Sept. 29, 14:30-15:30**

Fernando RUIZ RUIZ (Univ. Complutense de Madrid)

**Title: Geometric construction of D-branes**

Abstract: Motivated by the construction of noncommutative field theories with time and light-like noncommutativity, the construction of D-branes in Wess-Zumino Witten string backgrounds is considered. A geometric viewpoint, in which D-branes are defined as spacetime submanifolds swept by the string endpoints, is adopted. The starting point of the discussion is a matching equation for the WZW model chiral currents that poses a differential problem. The equation involves an in principle arbitrary linear map defined on the WZW Lie algebra. The equation is explicitly solved and conditions for the solutions to define D-branes are found.