Mini-Curso Temático

Jueves, 10 y 17 de noviembre de 2011

14:30 h., Aula Naranja (ICMat, Campus de Cantoblanco)

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The Operator Algebra Approach to Quantum Groups

Resumen:

Let $C(G)$ denote the continuous functions on a compact group G. By the Gelfand-Naimark Theorem, this C^*-algebra captures the topology on G. If we now define

$$\Gamma : C(G) \to C(G) \otimes_{min} C(G) \simeq C(G \times G) \quad \text{by} \quad \Gamma(f)(s,t) = f(st),$$

then Γ captures the group structure and, as $f((st)u) = f(s(tu))$, the $*$-homomorphism Γ will satisfy $(id \otimes \Gamma) \circ \Gamma = (\Gamma \otimes id) \circ \Gamma$, that is, Γ is a *comultiplication*. A *compact quantum group* is thus defined to be a pair (A, Γ) consisting of a unital C^*-algebra A and a comultiplication Γ satisfying certain conditions. The category of compact quantum groups also includes the reduced group C^*-algebras of discrete groups. However it also contains new and interesting examples of deformations of classical groups (e.g., Woronowicz's quantum $SU(2)$).

Another motivation for the analytic approach to quantum groups was the desire to generalize the Pontryagin duality theorem for locally compact abelian groups: If G is a locally compact abelian group, then $\hat{G} := \{\phi : G \to \mathbb{C} \mid \phi(g_1 g_2) = \phi(g_1)\phi(g_2)\}$ is again a locally compact abelian group and $\hat{\hat{G}}$ is canonically isomorphic to G. However, this fails for nonabelian groups. Vaes and Kustermans' *locally compact quantum groups* include the locally compact groups, their dual objects, and satisfy a Pontryagin duality theorem.

Compact and locally compact quantum groups will be defined, and the basics of their duality/representation theory will be described (concentrating mainly on the compact quantum group case), before briefly discussing some examples.