

Minicourse on Abelian integrals and limit cycles

Magdalena Caubergh (UAB)

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11:00-13:00: Lecture 1.- Method based on the computation of abelian integrals in the study of limit cycles.

15:30-17:30: Lecture 2.- Phenomenon of alien limit cycles and limit cycles escaping to infinity.

Resumen:

The aim is to give a short course (two lectures) on Abelian integrals in view of its relation to limit cycles of smooth planar vector fields that are nearly Hamiltonian. Limit cycles are isolated periodic orbits that, as traditionally, by the introduction of the Poincaré first return mapping are controlled by the zeroes of a smooth equation. When studying limit cycles that bifurcate from a Hamiltonian period annulus the first order approximation of this equation corresponds to the so-called related Abelian integral or first order Melnikov function. In the first lecture we recall the notion of Abelian integral in a general context and in relation to limit cycles. Besides we recall algorithms to compute Abelian integrals and higher order Melnikov functions (J.-P. Francoise) and to study their properties (Picard-Fuchs equation). We illustrate these algorithms and their use for bifurcations of limit cycles from the harmonic oscillator and the Bogdanov-Takens codimension 2 vector field. The approximation method based on Melnikov functions is very powerful and lead to precise results of the bifurcation diagram in case one restricts to limit cycles that bifurcate from the interior of the period annulus. In the second lecture we focus our attention to limit cycles that bifurcate from the boundary. By examples we point out that then often not all limit cycles are controlled by the related Abelian integral; this phenomenon is known as the presence of alien limit cycles. For instance, Abelian integrals do not detect limit cycles that come arbitrarily close to the singular boundary neither those that escape to infinity. Although the Abelian integral does not control all limit cycles in a direct way, we develop a practical method to detect these phenomena using besides the Abelian integral a generic criterion on the transition along the regular parts of the singular cycle. Part of this talk is based on joint work with F. Dumortier and R. Roussarie.

Referencias:

- [1] M. Caubergh and F. Dumortier, Hopf-Takens bifurcations and centers. J. Differential Equations 202 (2004) 1-31.
- [2] F. Dumortier and R. Roussarie, Abelian integrals and limit cycles. J. Differential Equations 227 (2006) 116-165.
- [3] S. Luca, F. Dumortier, M. Caubergh and R. Roussarie, Detecting alien limit cycles near a Hamiltonian 2-saddle cycle. Discrete Cont. Dynam. Sys. A 25 (2009) 1081-1108.