Asymptotics of orthogonal polynomials with respect to oscillatory weights

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Abstract

We present recent results on the asymptotic behavior and asymptotic zero distribution (as the degree and/or other parameters tend to $\infty$) of polynomials $p_n(x)$ that are orthogonal with respect to a weight function $w(x)$ that is oscillatory on the real axis. The two main examples will be $w(x) = e^{i\omega x}$ on $[-1, 1]$, where $\omega > 0$ is a real parameter, potentially large, and $w(x) = J_\nu(x)$ on $[0, \infty)$, where $J_\nu(x)$ is the Bessel function of order $\nu$. Because of the oscillatory nature of the weight function, the sequence of orthogonal polynomials is not guaranteed to exist or could be lacunary. The tools used for the analysis are logarithmic potential theory in the complex plane and the $S$-property, together with the Riemann-Hilbert formulation and the Deift-Zhou steepest descent method.

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References

